Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 4TE04EMT1 **Subject Name: Engineering Mathematics-IV**

Course Name: B.Tech(All) Date: 19/5/2015

Semester: IV Marks: 70

Time: 02:30 TO 05:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION – I

Q-1 (a) Write the formula for Weddle's Rules. [01]

(b) Evaluate $\Delta^n e^x$. [02]

(c) Solve: $(1+\Delta)(1-\nabla)=1$ [02]

(d) Show that the function $f(z)=|z|^2$ is differentiable only at origin. [02]

Q-2 (a) Show that the function $u(x,y)=y+e^x\cos y$ is harmonic function and determine their conjugate harmonic. [05]

(b) Find the image of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$.Also the regions graphically. [05]

(c) The function $w=\log z$ is analytic or not? [04]

OR

Q-2 (a) If $f(z)=u+iv$ is an analytic function of $z=x+iy$ And $u-v=e^x(\cos y-\sin y)+x+y$, Find $f(z)$. [05]

(b) Determine the bilinear transformation which maps the point $0,\infty,i$ to $\infty,1,0$. [05]

(c) Show that the function $w=z^{5/2}$ is satisfied C-R equation. [04]

Q-3 (a) Given $10\frac{dy}{dx}=x^2+y^2, y(0)=1$. Using Runge-Kutta method of forth order, find $y(0.2)$ with $h=0.1$. [05]

(b) Solve the following system of equations by Gauss-Seidel method. $x+10y+z=6,\quad x+y+10z=6,\quad 10x+y+z=6$. [05]

(c) Evaluate $\int_0^3 \frac{dx}{1+x}$ with $w=6$ by using Simson's $\frac{3}{8}$ rule and hence calculate $\log 2$. [04]

OR

Q-3 (a) Use Runge –Kutta method of third order to find the solution of the initial value problem $\frac{dy}{dx}=x+\sqrt{y}$; $y(0)=4$ for $x=0.2$ taking $h=0.1$ correct up to four decimal. [05]

(b) Using Taylor's series method, Find $y(0.2)$ if $y(x)$ satisfies $\frac{dy}{dx}=2y+3e^{x}$; $y(0)=0$ Compare the numerical solution with the exact solution. [05]

(c) Consider the following tabular value. [04]

X	50	100	150	200	250
y	618	724	805	906	1032

Determine $y(300)$

SECTION-II

Q-4 (a) Evaluate $\lim_{z \to i} \frac{z^2+1}{z-i}$. [01]

(b) If $f(z)=\begin{cases} \frac{z}{\bar{z}} & ,z\neq 0 \\ 0 & ,z=0 \end{cases}$ continuous at origin? [02]

(c) Sketch the Regions $|2z+1+i|<4$. [02]

(d) Find $\nabla \phi$ at $(1,-2,1)$ if $\phi = 3x^2y-y^3z^2$. [02]

Q-5 (a) Using the Fourier integral representation prove that [05]

$$\int_0^\infty \frac{\cos \omega x+\omega \sin \omega x}{1+\omega^2}d\omega = \begin{cases} 0 & ,x<0 \\ \frac{\pi}{2} & ,x=0 \\ \pi e^{-x} & ,x>0 \end{cases}$$

(b) Verify Green's theorem for $\oint_C \left(x^2-2xy\right)dx+\left(x^2y+3\right)dy$ where C is the boundary of the region bounded by the parabola $y=x^2$ and line $y=x$. [05]

(c) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ along the parabola $y^2=x$ between the points $(0,0)$ and $(1,1)$ where $\bar{F}=x^2\hat{i}+xy\hat{j}$ [04]

OR

Q-5 (a) Find the Fourier transformation of $e^{-x^2a^2}$ $,a>0$ [05]

(b) Evaluate the Stokes theorem $\oint_C (4y dx + 2z dy + 6y dz)$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = 6z$ and the plane $z = x + 3$ [05]

(c) Find $curl \cdot curl\ \bar{A} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ at point $(1, 0, 2)$. [04]

Q-6 (a) Given

X	10	20	30	40	50
y	600	512	439	346	243

Using sterling's formula find y_{35} [05]

(b) Use Langrage's interpolation formula to find the value of y when $x = 10$ if the value of x and y are given below [05]

X	5	6	9	10
Y	12	13	14	16

(c) Evaluate $I = \int_2^3 \frac{\cos 2x}{1 + \sin x} dx$ Using Gaussian two point and three point formula. [04]

OR

Q-6 (a) Prove that [05]

(1) $\Delta = e^{hD} - 1$.

(2) $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$.

(b) Using Euler modified method, solve $\frac{dy}{dx} = \log(x + y)$; $y(1) = 2$ at $x = 1.2$ and $x = 1.4$,taking $h = 0.2$ correct up to 4-decimal [05]

(c) Solve: $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$ by Gauss elimination method. [04]